Lemma 0.1. *Suppose $\mathcal{H}_i$ separates a from all the P_j except possibly P_i. Then (if P_i are constructed by the regularity lemma) all rainbow simplices contain a. This is equivalent to a belonging to the unique bounded region of the arrangement $\{\mathcal{H}_i\}$.*

Suppose $\mathcal{H}_i = \{x : xv_i = c_i\}$, $1 \le i \le n$.
Could a belong to the region $I \subseteq [n]$?
Then $a \in \bigcap_{i \in I} \mathcal{H}_i^+ \bigcap_{i \notin I} \mathcal{H}_i^-$.
Lookat opposite region.
Assume $v_1 + \cdots + v_n = 0$.