Let G be a graph such that $d(x) \geq n/2$ for each $x \in V(G)$, where $n = |V(G)|$. Then G has a Hamiltonian cycle.

We prove this by finding a Hamiltonian path. This is sufficient: suppose $x_1 \ldots x_n$ is a Hamiltonian path in G and consider the two sets $S = \{i : x_1 x_i \in E(G)\}$ and $T = \{i : x_{i-1} x_n\}$. We can assume $n \notin S$ and $2 \notin T$. Then there is an $i \in S \cap T$, which gives us a Hamiltonian cycle $x_1 x_i \to x_n x_{i-1} \leftarrow x_1$.

Now we find a Hamiltonian path. Clearly, there is a path of length $\geq n/2$, since all degrees are $\geq n/2$. Let $P = x_1 \ldots x_r$ be a longest path, where $r \geq n/2$. Let S be the set of i such that x_i is connected to some vertex outside P. We may assume that $1, r \notin S$. It is now sufficient to consider the following picture: A vertex in S adjacent to a vertex outside S on P, together with their edges to vertices in P, with special attention to x_1 and x_r.